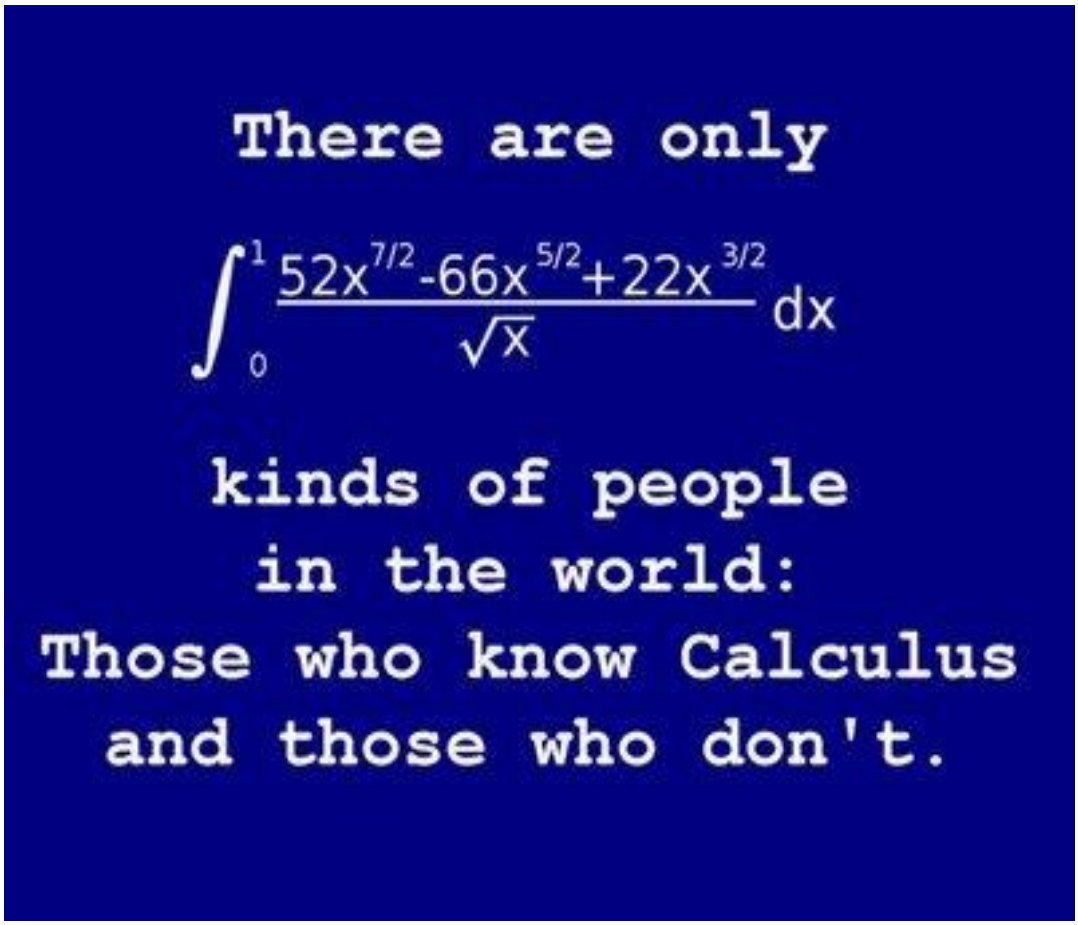
AP Calculus BC

Summer Assignment



Welcome to **AP Calculus BC**! This review assignment is designed to refresh your Algebra I, Algebra II, Pre-Calculus, and Analytic Geometry skills. It includes information that was taught in previous courses and will be used throughout the upcoming school year. As you prepare, you may need to seek help by accessing the suggested resources or links provided.

NOTE: *Look out for a "Trig for Calculus Boot Camp" CANVAS course on your dashboard. Through it, you will be invited to participate in an on-campus class this summer that will help you solidify the prerequisites needed for your class next school year. Stay tuned for registration information.*

IMPORTANT: Read this page first...

INSTRUCTIONS

1. Complete all sections and problems in this packet on your own.
2. Make sure to show your work to earn credit.
3. Try to complete the questions without a graphing calculator.

PACING

You should pace yourself to work on this assignment at least a few hours a week leading up to the start of school in September. If you complete the packet at the end of June or early in July, it will not be very helpful in preparation for the start of school. Also, it will not be helpful if you try to complete the entire packet a night or two before school starts. Pace yourself by setting a calendar reminder and scheduling blocks of time to focus on this assignment as you prepare to return to school in September.

GRADING

- On the first day of school, your math teacher will check for full completion of this Summer Assignment and the supporting work for your responses (no work = no credit). This part will be weighted at 50% - this is the grade that represents your effort and following of directions.
- Your teacher will then review the assignment and provide remediation as needed.
- Upon completion of your teacher's review, you will be given an assessment (a "test") based on the topics covered in this assignment. This assessment will be weighted at 50% - this is the grade that represents your mastery of the skills.
- The two weighted scores combined will count as one project grade for the 1st trimester.
- Acceptance of late assignments will be limited and subject to point deductions.

We are looking forward to meeting you in September.
Go Bulldogs!

RESOURCES & REFERENCE MATERIALS

http://www.calcchat.com/book/Calculus-9e/
http://apcentral.collegeboard.com/apc/public/courses/teachers_corner/2178.html
http://www.calculus.org/
http://cow.math.temple.edu/
http://www.mathsisfun.com/calculus/
http://www.wolframalpha.com/widgets/view.jsp?id=dc816cd78d306d7bda61f6facf5f17f7
http://www.wolframalpha.com/widgets/view.jsp?id=c44e503833b64e9f27197a484f4257c

1. Use the **definition of a derivative** to find $f'(x)$ if $f(x) = \sqrt{x+1}$

Related Rates

2. You are inflating a spherical balloon at a rate of 7 cm^3/sec. How fast is its radius increasing when the radius is 4 cm?

3. Water is poured into a conical container at the rate of 10 cm^3/sec. The cone points directly down, and it has a height of 30 cm and a base radius of 10 cm. How fast is the water level rising when the water is 4 cm deep (at its deepest point)?

4. A ladder 13 meters long rests on horizontal ground and leans against a vertical wall. The foot of the ladder is pulled away from the wall at the rate of 0.6 m/sec. How fast is the top sliding down the wall when the foot of the ladder is 5 m from the wall?

5. A road running north to south crosses a road going east to west at the point P. Car A is driving north along the first road, and car B is driving east along the second road. At a particular time car A is 10 kilometers to the north of P and traveling at 80 km/hr, while car B is 15 kilometers to the east of P and traveling at 100 km/hr. How fast is the distance between the two cars changing?

Find the derivative of each function:

6. $f(x) = 6\sqrt{x} + 3\sqrt[3]{x}$

7. $f(t) = 2t^5 cost$

8. $f(x) = x2^x$

9. $g(x) = 4xe^x - cotx$

10. $f(x) = 5xcos(2x + 5)$

11. $f(x) = sin^3(\sqrt{x})$

12. $ln(x + y) = x$

13. $f(x) = x\ arcsinx$

14. $f(x) = arctan(x^2 - 8)$

15. Define $f(x) = \frac{|x-1|}{x}$

(a) Show that $f(x)$ is continuous at $x = 2$.

(b) Where on the interval [-2,2] is f discontinuous? Show the work that leads to your conclusion.

(c) Classify the discontinuities in part (b) as removable or nonremovable.

Find the limit (if it exists). If the limit does not exist, explain why.

16. $\lim_{y\to 0} \frac{(x+y)^3 - x^3}{y}$

17. $\lim_{x\to 6}(x-2)(x-2)^2$

18. $\lim_{x\to 0}\frac{1-cosx}{sinx}$

19. $\lim_{x\to\infty}\frac{2x^2}{3x^2+5}$

20. $\lim_{x\to\infty}\left(8+\frac{1}{x}\right)$

Free-Falling Object, for 21 and 22 use the position function $s(t) = -4.9t^2 + 250$ which gives the height (in meters) of an object that has fallen from a height of 250 meters.

21. Find the velocity of the object when $t = 4$.

22. At what velocity will the object impact the ground?

Find the one-sided limit (if it exists)

23. $\lim_{x\to 0^+}\frac{sin4x}{5x}$

24. $\lim_{x\to 0^-}\frac{cos^2x}{x}$

25. $\lim_{x\to 3^-}\frac{|x-3|}{x-3}$

26. Find the equation of the tangent to the graph of $f(x) = 3x - 5cos2x \; at \; x = 0.$

27. Suppose $g(0) = 4, g'(0) = 8, and \; g''(0) = -12. If \; h(x) = \sqrt{g(x)}$ $. What \; is \; h''(0)?$

28.

The function f is defined as $f(x) = \dfrac{x^2 + 5x + 6}{2x^2 + 7x + 3}$.

(a) State the value(s) of x for which f is not continuous.

(b) Evaluate $\lim_{x \to -3} f(x)$. Justify your answer.

(c) State the equation(s) for the vertical asymptote(s) of the graph of $y = f(x)$.

(d) State the equation(s) for the horizontal asymptote(s) of the graph of $y = f(x)$. Show the work that leads to your answer.

29.

Let a and b represent real numbers. Define

$$f(x) = \begin{cases} ax^2 + x - b, & \text{if } x \leq 2 \\ ax + b, & \text{if } 2 < x < 5. \\ 2ax - 7, & \text{if } x \geq 5 \end{cases}$$

(a) Find the values of a and b such that f is continuous on the entire real number line.

(b) Evaluate $\lim_{x \to 3} f(x)$.

(c) Let $g(x) = \dfrac{f(x)}{x - 1}$. Evaluate $\lim_{x \to 1} g(x)$.

30.

Let f be a function defined by

$$f(x) = \begin{cases} e^{2x}, & x \leq 0 \\ 4 - 3\cos x, & x > 0 \end{cases}.$$

(a) Find $\lim_{x \to -1} f(x)$.

(b) Show that f is continuous at $x = 0$.

(c) Find $\lim_{x \to -\infty} f(x)$.

31.

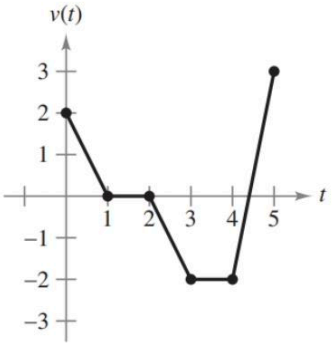
Given: $g(x) = f(x) \cdot \tan x + kx$, where k is a real number. f is differentiable for all x; $f(\pi/4) = 4$; $f'(\pi/4) = -2$.

(a) For what values of x, if any, in the interval $0 < x < 2\pi$ will the derivative of g fail to exist? Justify your answer.

(b) If $g'\left(\frac{\pi}{4}\right) = 6$, find the value of k.

32.

The figure below shows the graph of the velocity, in feet per second, for a particle moving along the line $x = 4$.



(a) During which time interval(s) is the particle:

(i) moving upward?

(ii) moving downward?

(iii) at rest?

(b) What is the acceleration of the particle at

(i) $t = 0.75$ and

(ii) $t = 4.2$? Be sure to include units.

33.

Let f be a function defined by

$$f(x) = \begin{cases} e^{2x}, & x \le 0 \\ 4 - 3\cos x, & x > 0 \end{cases}.$$

(a) Find $\lim_{x \to -1} f(x)$.

(b) Show that f is continuous at $x = 0$.

(c) Find $\lim_{x \to -\infty} f(x)$.

34.

Given:

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
2	-3	1	5	-2
5	4	7	-1	2

(a) If $h(x) = \dfrac{f(x)}{g(x)}$, find $h'(2)$.

(b) If $j(x) = f(g(x))$, find $j'(2)$.

(c) If $k(x) = \sqrt{f(x)}$, find $k'(5)$.

35. What is the equation of the normal line (perpendicular to tangent line) to

$f(x) = 3x - 5cos2x$ at $x = 0$?

Multiple-choice questions

36. A square is inscribed in a circle. How fast is the area of the square changing when the area of the circle is increasing at the rate of 1 in^2/min?

a) 1 $\frac{in^2}{min}$

b) $\frac{1}{2}\frac{in^2}{min}$

c) $\frac{2}{\pi}\frac{in^2}{min}$

d) 1 $\frac{in^2}{min}$

e) $\frac{\pi}{2}\frac{in^2}{min}$

37. What is the slope of the line tangent to the curve $y^3 + x^2y^2 - 3x^3 = 9$ at the point (1,2)?

a) $\frac{1}{16}$

b) $\frac{1}{8}$

c) $\frac{1}{4}$

d) $-\frac{1}{4}$

e) $-\frac{1}{8}$

38.

The position $s(t)$ of a particle moving along the x-axis at time t is given by $s(t) = -t^3 + 2t^2 + \frac{3}{2}$, where s is measured in meters and t is measured in seconds. At what time is the particle's instantaneous velocity equal to its average velocity on the interval $[0, 4]$?

(A) 1.097 seconds (B) 2 seconds

(C) 2.333 seconds (D) 2.431 seconds

39.

. The table shows the position $s(t)$ of a particle that moves along a straight line at several times t, where t is measured in seconds and s is measured in meters.

t	2.0	2.7	3.2	3.8
$s(t)$	5.2	7.8	10.6	12.2

Which of the following best estimates the velocity of the particle at $t = 3$?

(A) 3.7 m/sec (B) 3.9 m/sec

(C) 5.6 m/sec (D) 7.8 m/sec

40.

If $h(x) = |2x - 5|$, which of the following is true?

(A) h is continuous but is not differentiable at $x = \frac{5}{2}$.

(B) h is not continuous but is differentiable at $x = \frac{5}{2}$.

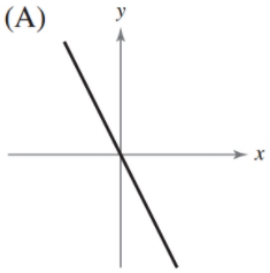
(C) h is continuous and differentiable at $x = \frac{5}{2}$.

(D) h is neither continuous nor differentiable at $x = \frac{5}{2}$.

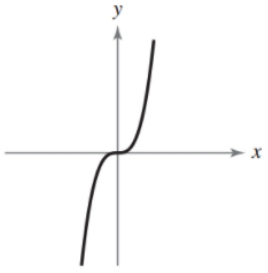
41.

Which graph shows a function whose derivative is always negative?

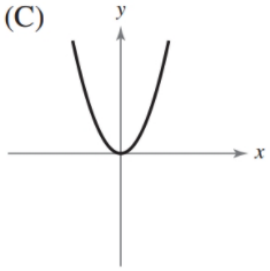
(A)



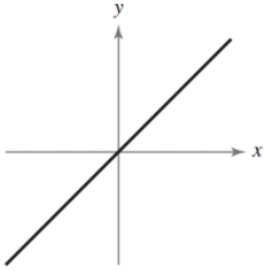
(B)



(C)



(D)



42.

What is an equation of the tangent line to the graph of $f(x) = 4e^x - x + 6$ at $(0, 10)$?

(A) $y = 4x + 10$

(B) $y = 4x - 10$

(C) $y = 10x - 4$

(D) $y = 3x + 10$

43.

If $y = \dfrac{6x^4 - 3x^5 + 5x^3}{x^3}$, then $\dfrac{d^2y}{dx^2} =$

(A) $6 - 6x.$

(B) $6.$

(C) $6x.$

(D) $-6.$

44.

. If $2y^3 - 3xy + x^2 = 4$, then $\frac{dy}{dx} =$

(A) $-\frac{2x}{6y^2 - 3}$.

(B) $\frac{2x - 3y}{3x - 6y^2}$.

(C) $\frac{2x - 3}{6y^2}$.

(D) $-\frac{2x}{6y^2 - 3x}$.

45.

If $\lim_{x \to 5} f(x) = 10$ and $\lim_{x \to 5} g(x) = 1$, what is the limit of $\lim_{x \to 5} [5f(x) - g(x)]$?

(A) 9

(B) 15

(C) 45

(D) 49